Safety-Certified Consensus Control of Multi-Agent Systems Based on Finite-Time Control Barrier Function

Wentao Wu¹,², Yibo Zhang², Weidong Zhang²,³, Di Wu²,³
1. SJTU Sanya Yazhou Bay Institute of Deepsea Science and Technology, Hainan, 572024, China
2. Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China
3. School of Information and Communication Engineering, Hainan University, Haikou 570228, Hainan, China
E-mail: {wdzhang, zhang297}@sjtu.edu.cn

Abstract: This paper investigates the consensus control of second-order multi-agent systems in the presence of stationary obstacles. Every agent is faced with unknown model uncertainties. Based on the finite-time control barrier function (FTCBF), a safety-certified consensus control method is proposed to achieve collision-free consensus formation. First, a nominal virtual control law is designed to generate the predefined formation. An FTCBF-based state set is constructed to guarantee the safety of multi-agent systems. Next, a finite-time observer with robust exact differentiator is devised to recover unknown model uncertainties. By using the estimated information, we develop an anti-disturbance control law to track the optimal signal. Finally, error signals of the closed-loop system are proved to be uniformly ultimately bounded, and the multi-agent system can be ensured to be safety-certified. A simulation example is employed to demonstrate the effectiveness of the proposed safety-certified consensus control method.

Key Words: Multi-agent system, consensus control, finite-time control barrier function, finite-time observer

1 Introduction

Over the past years, leader-following consensus control of multi-agent systems has aroused increasing attention from various communities and has been applied in many fields, such as unmanned aerial vehicles, unmanned mobile robots, unmanned surface vehicles, and unmanned underwater vehicles [1–6]. Different from leader-less consensus control [7–9], leader-following consensus control aims to drive all controlled objects to achieve a common motion in the presence of leaders under the distributed communication. [9–13]. Leader-following consensus control guide by multiple leaders is also considered as containment control [14–16]. According to the property of leaders, leader-following control can be further divided into containment tracking guided by time-varying trajectories [9–11] and containment maneuvering guided by parameterized paths [12, 13]. Note that these existing containment control methods are under ideal safety conditions [9–16].

Safety is an important requirement when controlling multi-agent systems. Safety conditions include collision avoidance, connectivity preservation, communication delay, states and input constraints. Control barrier functions (CBFs) have been proposed as an efficient tool to design safety-critical controllers [17–19]. The key point of CBFs is to map constrained states onto constrained inputs by means of forwarding invariance of sets under safety conditions. Recently, many CBFs-based safety-critical control methods have been proposed for nonlinear systems [17–19]. However, these CBFs-based safety-critical control methods are centralized.

Motivated by the above observations, we propose a safety-certified distributed control method for leader-following consensus of a class of second-order nonlinear multi-agent systems based on finite-time control barrier function (FTCBF). The dynamics of followers is guided by a parameterized path and in the presence of unknown nonlinearities and stationary obstacles. At first, we design a nominal virtual control law to generate the predefined formation. Next, we construct an FTCBF-based state constraint set to guarantee safety among multi-agent systems. Then, a Levant finite-time observer is utilized to recover unknown model uncertainties. Finally, an anti-disturbance control law is constructed by using the estimated information. All error signals in the closed-loop is proved to be uniformly ultimately bounded, and the safety of multi-agent systems is guaranteed. In contrast to the existing leader-following consensus control methods in [9–16], the proposed consensus control method herein is not only collision-free but also finite-time safety-critical.

This paper is organized as follows. Section II gives problem formulation. Section III presents the controller design. Section IV shows the stability analysis. Section V provides the simulation results to demonstrate the effectiveness of the proposed method. Section V concludes this paper.

2 Preliminaries and Problem formulation

2.1 Notations

Define the following notations used in this paper. \( \mathbb{R} \), \( \mathbb{R}^+ \), and \( \mathbb{R}^n \) is a real set, a positive real set, and an \( n \)-dimensional vector set, respectively. \( \| \cdot \| \) denotes the Euclidean norm of a vector. For \( s = [s_1, \ldots, s_n]^T \in \mathbb{R}^n \) and \( 0 < \varepsilon < 1 \), \( \sigma^\varepsilon(s) \) is defined as \( \sigma^\varepsilon(s) = [\text{sign}(s_1)|s_1|^\varepsilon, \ldots, \text{sign}(s_n)|s_n|^\varepsilon]^T \), where \( \text{sign}(\cdot) \) is a signum function.

4661
2.2 Problem formulation

Consider a multi-agent system containing \( N \) second-order agents. Here, the dynamics of the \( i \)-th agent is expressed as

\[
\dot{x}_i(t) = f_i(x_i, t) + u_i(t),
\]

where \( x_i(t) \in \mathbb{R}^2 \) and \( u_i(t) \in \mathbb{R}^2 \) are the state of the agent with \( x_i(t) = [x_{i1}(t), x_{i2}(t)]^T \). \( f_i(x_i, t) \in \mathbb{R}^2 \) denotes the unknown model uncertainty. \( u_i(t) \in \mathbb{R}^2 \) presents the actual control input.

Consider a virtual leader moving along the smooth parametrized path \( x_0(\theta) \in \mathbb{R}^2 \), where \( \theta \) is a parameter variable. A communication graph \( G \) is defined as \( G = (\mathcal{V}, \mathcal{E}) \) with \( \mathcal{V} \) and \( \mathcal{E} \) being a node and an edge set, respectively. \( \mathcal{V} = \{ \mathcal{V}_F, \mathcal{V}_L \} \) is the set of agents and the virtual leader. \( \mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} \} \) is used to depict the communication links of agents and the virtual leader. The following assumptions are used in this paper.

\textbf{Assumption 1: } The first-order partial derivative of the given signal \( x_0(\theta) \) is bounded and satisfies \( \| x_0^\theta \| \leq \bar{x}_0^\theta \), with \( x_0^\theta = \partial x_0 / \partial \theta \) and \( x_0^\theta \in \mathbb{R}^2 \).

\textbf{Assumption 2: } For the communication topology of multiple agents and one virtual leader, there is at least one agent that can obtain the information of the virtual leader.

The safety-certified consensus control of second-order multi-agent system need to satisfy the following objectives.

- **Consensus objective:** Drive each agent to follow the virtual leader and hold a predefined deviation such that

\[
\lim_{t \to \infty} \| x_i(t) - x_0(\theta(t)) - x_o \| \leq \delta_1,
\]

where \( x_o \in \mathbb{R}^2 \) is a predefined deviation from the virtual leader. \( \delta_1 \) is a positive constant.

- **Dynamic objective:** Force the parameter dynamics \( \dot{\theta}(t) \) to converge to the expected speed such that

\[
\lim_{t \to \infty} \| \dot{\theta}(t) - \nu_s \| \leq \delta_2,
\]

where \( \nu_s \in \mathbb{R} \) is a predefined speed. \( \delta_2 > 0 \) is a small constant.

- **Safety objective:** Ensure no collision between agents and static obstacles such that

\[
\| x_i - x_o \| \geq D_o + \rho_o, \quad o = 1, \ldots, N_o,
\]

where \( x_o \in \mathbb{R}^2 \) is the position of static obstacles. \( N_o > 0 \) is the total number of obstacles. \( \rho_o > 0 \) denotes the radius of the \( o \)-th obstacle. \( D_o > 0 \) stands for the preset safe distance.

3 Controller design

In this section, a distributed control law is designed to achieve the consensus formation. To ensure no collisions between agents and static obstacles, a safety-certified region and an admissible state set are constructed by using the finite-time control barrier function (FTCBF). Based on robust exact differentiator (RED), a finite-time observer is devised to estimate unknown uncertainties of agents. Then, an actual control law is developed.

\textbf{Step 1: } Define a consensus tracking error \( e_{i1} \) as follows

\[
e_{i1} = \sum_{j=1}^{N} a_{ij}(x_{i1} - x_{j1} - x_{ij0}) + a_{i0}(x_{i1} - x_0(\theta) - x_{i0}),
\]

where if the \( i \)-th agent can receive the information from the \( j \)-th agent, \( a_{ij} = 1, i \neq j, j = 1, \ldots, N \); otherwise, \( a_{ij} = 0 \). If the \( i \)-th agent can obtain the information from the virtual leader, \( a_{i0} = 1 \); otherwise, \( a_{i0} = 0 \). \( x_{ij0} = x_o - x_{j0} \).

Taking the derivative of \( e_{i1} \) along (1), one has

\[
\dot{e}_{i1} = d_i x_{i2} - \sum_{j=1}^{N} a_{ij} x_{j2} - a_{i0} x_{i0}^\theta \dot{\theta},
\]

where \( d_i = \sum_{j=1}^{N} a_{ij} + a_{i0} \).

To stabilize the error dynamics in (6), we design a distributed control law as follows

\[
\alpha_i = \frac{1}{d_i} \left( -k_{i1} e_{i1} + \sum_{j=1}^{N} a_{ij} x_{j2} + a_{i0} x_{i0}^\theta \nu_s \right),
\]

where \( k_{i1} \) is a positive gain constant.

To coordinate agents and reference signal, the update law is designed as

\[
\dot{\theta} = -\lambda \left( \theta + \epsilon \sum_{j=1}^{N} a_{i0} (x_{i0}^\theta)^T e_{i1} \right).
\]

To avoid the collision between agents ans \( N_o \), static obstacles, a FTCBF \( h_{io}(x_i) \) is constructed as follows

\[
h_{io}(x_i) = \| x_{i1} - x_o \|^2 - \rho_o^2,
\]

where \( o = 1, \ldots, N_o \).

Based on the designed FTCBF \( h_{io}(x_i) \), a safety-certified motion region for the \( i \)-th agent can be described as

\[
C_{io}(x_{i1}) = \{ x_{i1} \in \mathbb{R}^2 : h_{io}(x_i) \geq 0 \}.
\]

To ensure that agents can go into the safety region in finite time, an admissible state space is developed as [20]

\[
S_{io}(x_{i2}) = \{ x_{i2} \in \mathbb{R}^2 : h_{io}(x_i) \geq -\zeta_i \text{sgn}(h_{io}) \}.
\]

where \( \zeta_i \geq 0 \) and \( 0 < \epsilon_i < 1 \).

In order to unify the tracking performance and safety requirement, a distributed quadratic optimization is employed as follows

\[
x_{i2}^* = \arg \min_{x_{i2} \in \mathbb{R}^2} J_i(\alpha_i) = \| x_{i2} - \alpha_i \|^2
\]

s.t. \( x_{i2} \in S_{io}(x_{i2}), \quad o = 1, \ldots, N_o \).

Noting that the quadratic optimization with multiple constraints can be solved by neurodynamic optimization such as projection neural network [21] and recurrent neural network [22]. It can obtain \( \| x_{i2} - \alpha_i \| \leq \bar{\alpha} \) with \( \bar{\alpha} \) being a positive constant.
Step 2: To acquire the smooth property and the derivative of $x_{i2}$, a linear tracking differentiator (LTD) is devised as

$$
\begin{align*}
\dot{q}_f &= q_{f}^d, \\
\dot{q}_{f}^d &= -\mu q_{i}^d (q_f - x_{i2}) - 2\mu q_f q_{i}^d,
\end{align*}
$$

(13)

where $q_f$ and $q_{f}^d$ represents the estimated information of $x_{i2}$ and $\dot{x}_{i2}$. According to the stability analysis from [23], it can be concluded that the filtered signals $q_f$ and $q_{f}^d$ can converge to the neighbor region of $x_{i2}$ and $\dot{x}_{i2}$, i.e., satisfying $||q_f - x_{i2}|| \leq \tilde{q}_i$ and $||q_{f}^d - \dot{x}_{i2}|| \leq \tilde{q}_f$ with $q_f$ and $q_{f}^d$ being the positive constants.

Step 3: Define the estimated errors $\tilde{x}_{i2} = x_{i2} - x_{i2}$ and $\tilde{f}_i = \dot{f}_i - f_i$, where $\dot{x}_{i2}$ and $f_i$ are the estimations of $x_{i2}$ and $f_i$, respectively. Then, a finite-time observer based on robust exact differentiator (RED) is developed as follows [24]

$$
\begin{align*}
\dot{\tilde{x}}_{i2} &= -\gamma_1 L_i^2 \text{sgn}^2(\tilde{x}_{i2}) + \tilde{f}_i + u_i, \\
\dot{\tilde{f}}_i &= -\gamma_2 L_i \text{sgn}^2(\tilde{x}_{i2}),
\end{align*}
$$

(14)

where $\gamma_1$ and $\gamma_2$ are the designed positive constants. $L_i$ is a scaling factor. It is assumed that the first-order derivative of $f_i$ is bounded. According to [25], the first-order observer (14) is finite-time stable, which means the estimated error $\tilde{f}_i$ is bounded and satisfies $||\tilde{f}_i|| \leq \tilde{f}_i \in \mathbb{R}^+$. Letting the tracking error $e_{i2} = x_{i2} - q_f$, we differentiate it along (1) as

$$
\dot{e}_{i2} = f_i + u_i - q_{f}^d.
$$

(15)

To converge the error $e_{i2}$, a observer-based control law is designed as follows

$$
u_i = -k_{i2} e_{i2} - \tilde{f}_i + q_f^d - d_i e_{i1}.
$$

(16)

where $k_{i2}$ is the designed positive constant.

4 Safety and Stability analysis

The safety analysis of multi-agent systems is given by the following lemma. Lemma 1: Given a second-order agent expressed as (1), if the state variable $x_{i2}$ belongs to the safety-certified state set defined by (11) for all agents and $x_{i2} \in S_{i0}(x_{i1})$, $i = 1, ..., N$, the closed-loop multi-agent system is safety-certified.

Proof: According to [21, 22], the state $x_{i2}$ can converge to the optimal solution $x_{i2}$ and meet the collision-free condition. From [20], it gets that the set $C_{i0}(x_{i1})$ is forward invariant by using the optimal state $x_{i2}$, i.e. the set $C_{i0}(x_{i1})$ is safe. It also shows that the agent state $x_{i1}(t)$ will stay in the set $C_{i0}(x_{i1})$ for $t > 0$ if it satisfies $x_{i1}(t_0) \in C_{i0}(x_{i1})$, $i = 1, ..., N$. Thus, it is ensured that the closed-loop multi-agent systems is safety certified.

Recalling error dynamics of the closed-loop system by using the equations (6), (7), (8), (15) and (16), we have

$$
\begin{align*}
\dot{e}_{i1} &= -k_{i1} e_{i1} + d_i z_i + a_i \varphi_0 \vartheta, \\
\dot{e}_{i2} &= -k_{i2} e_{i2} - \tilde{f}_i - d_i e_{i1}, \\
\dot{\vartheta} &= -\lambda (\vartheta + \varepsilon \sum_{i=1}^{N} a_i (x_0^i)^T e_{i1}),
\end{align*}
$$

(17)

where $z_i = x_{i2} - \alpha_i$.

The stability of the closed-loop system is shown via the following theorem.

Theorem 1: Consider the multi-agent system expressed as (1) in presence to multiple static obstacles with the distributed control law (7), the parameter update law (8), the admissible state space (11), the optimal control law (12), the LTD (13), the Levant observer (14) and the actual control law control law (16). Under Assumptions 1 and 2, all error signals of the proposed closed-loop system are uniformly ultimately bounded, and the multi-agent system is safe.

Proof: Construct a candidate Lyapunov function as

$$V = \frac{1}{2} \sum_{i=1}^{N} (e_{i1}^T e_{i1} + e_{i2}^T e_{i2}) + \frac{\vartheta^2}{2 \lambda \varepsilon}.
$$

(18)

Taking the time derivative of $V$ along the dynamics (17), one has

$$
\begin{align*}
\dot{V} &= \sum_{i=1}^{N} \{e_{i1}^T (-k_{i1} e_{i1} + d_i z_i + a_i \varphi_0 \vartheta) + e_{i2}^T (-\tilde{f}_i - k_{i2} e_{i2})\} \\
&- k_{i2} e_{i2} e_{i1} - d_i e_{i1} - \vartheta + \varepsilon \sum_{i=1}^{N} a_i (x_0^i)^T e_{i1}
\end{align*}
$$

(19)

$$
\begin{align*}
&= \sum_{i=1}^{N} \{k_{i1} ||e_{i1}||^2 - k_{i2} ||e_{i2}||^2 + d_i e_{i1}^T z_i \} \\
&- e_{i2}^T \tilde{f}_i - d_i e_{i2} e_{i1} - \vartheta^2 / \varepsilon.
\end{align*}
$$

Since $z_i = e_{i2} + q_{f}^d - x_{i2} + x_{i2} - \alpha_i$, it renders

$$
\begin{align*}
\dot{V} &\leq \sum_{i=1}^{N} \{k_{i1} ||e_{i1}||^2 - k_{i2} ||e_{i2}||^2 + \tilde{f}_i ||e_{i2}|| \} \\
&+ \vartheta + \varepsilon \sum_{i=1}^{N} a_i (x_0^i)^T e_{i1} - \vartheta^2 / \varepsilon.
\end{align*}
$$

(20)

Let $c_1 = \min \{k_{i1}, k_{i2}\}$, $c_2 = \max \{d_i (\alpha + \vartheta) / \tilde{f}_i\}$, $k = 1, ..., N$. The derivative of $V$ can be rewritten as

$$
\dot{V} \leq - c_1 \sum_{i=1}^{N} (||e_{i1}||^2 + ||e_{i2}||^2) - \vartheta^2 / \varepsilon
$$

(21)

Define $E = [e_{1}^T, e_{2}^T, \vartheta]^T$ with $e_1 = [e_{11}, ..., e_{N1}]^T$ and $e_2 = [e_{12}, ..., e_{N2}]^T$. The inequality (21) is presented as

$$
\dot{V} \leq - c'_1 ||E||^2 + c_2 ||E||,
$$

(22)

where $c'_1 = \min \{c_1, 1 / \varepsilon\}$.

For $||E|| \geq c_2 / (c_1 \varepsilon)$, it can obtain

$$
\dot{V} \leq - c'_1 (1 - \varepsilon) ||E||^2.
$$

(23)

Therefore, it can be concluded that all errors of the closed-loop system is uniformly ultimately bounded, and it satisfies

$$
||E(t)|| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \max \{||E(t_0)|| e^{-\beta (t-t_0)}, \frac{c_2}{c_1 \varepsilon} \}
$$

(24)

for $t \geq t_0$, where $P = \text{diag} \{1, 1 / \lambda \varepsilon\}$ and $\beta = 2c_1 (1 - \vartheta) / \lambda_{\max}(P)$.
5 Simulation results

In this section, simulation results are provided to demonstrate the effectiveness of safety-certified consensus control method. One virtual leader, five agents and four circular obstacles are employed in this simulation, and Fig. 1 describes their communication topology.

![Fig. 1: The safety-certified containment formation.](image1)

In this simulation, uncertainty terms of these agents are set as $f_i = [0.2 \sin(x_{i,1}) \cos(x_{i,1}), 0.4 \sin(\pi/3 + x_{i,2}) \cos(x_{i,2})]^T$, $i = 1, ..., 5$. Their initial status are set as $x_1 = [-2, 0, 0, 0]^T$, $x_2 = [-4, -4, 0, 0]^T$, $x_3 = [-6, -8, 0, 0]^T$, $x_4 = [-8, -12, 0, 0]^T$, $x_5 = [-10, -16, 0, 0]^T$, respectively. We select the position and radius of static circular obstacles as $x_1 = [12, 18]^T$, $x_2 = [30, 32]^T$, $x_3 = [50, 46]^T$, $\rho_1 = 2$, $\rho_2 = 3$, $\rho_3 = 4$, respectively. Parameters of the proposed method are selected as $x_{10} = [0, 0]^T$, $x_{20} = [-8, 0]^T$, $x_{30} = [0, -8]^T$, $x_{40} = [-16, 0]^T$, $x_{50} = [0, -16]^T$, $D_s = 1$, $\nu_s = 0.5$, $k_{i1} = 0.8$, $\lambda = 10$, $\epsilon = 10$, $\zeta_i = 1$, $\epsilon_i = 0.8$, $\mu_i = 10$, $\gamma_{i1} = 2.12$, $\gamma_{i2} = 1.1$, $L_i = 3.5$, $k_{i2} = 10$.

Simulation results are shown in Fig. 2 - Fig. 5. Specifically, Fig. 2 depicts the actual trajectories of five agents in the presence of three circular obstacles. It can also be seen that every agent avoids all static obstacles while trying to hold consensus with other agents. The consensus tracking errors of all agents are presented by Fig. 3, which reflects that errors are able to converge to small neighbors of the origin in finite time. Fig. 4 describes the varying curves of FTCBFs of five agents. It can be concluded that the multi-agent systems is safety-certified. Fig. 5 gives the real uncertainties and the estimated ones from the proposed observer (14). It shows that the RED-based observer is capable to recover the unknown uncertainties.

![Fig. 2: The safety-certified containment formation.](image2)

![Fig. 3: The tracking errors of five agents.](image3)

![Fig. 4: The varying curve of finite-time control barrier function of five agents.](image4)

![Fig. 5: The model uncertainties of agent 1.](image5)
Fig. 6: The actual state of five agents.

6 Conclusion

This paper investigated the safety-certified consensus control of second-order multi-agent systems in presence of stationary obstacles. Each agent is also faced with the model uncertainties. First, a distributed virtual control law is proposed based on backstepping principle. The constructed FTCBFS can guarantee the safety of multi-agent systems. Next, a finite-time observer is devised to recover unknown model uncertainties. Finally, all errors of the closed-loop system are proved to be uniformly ultimately bounded, and the safety of multi-agent systems is able to be ensured. Simulation results demonstrate the effectiveness of the proposed safety-certified consensus control method.

References


