Proceedings of the 41st Chinese Control Conference
July 25-27, 2022, Hefei, China

Safety-Certified Consensus Control of Multi-Agent Systems Based on Finite-Time Control Barrier Function

Wentao Wu[1,2], Yibo Zhang[2], Weidong Zhang[2,3], Di Wu[2,3]
1. SJTU Sanya Yazhou Bay Institute of Deepsea Science and Technology, Hainan, 572024, China
2. Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China
3. School of Information and Communication Engineering, Hainan University, Haikou 570228, Hainan, China
E-mail: {wdzhang, zhang297}@sjtu.edu.cn

Abstract: This paper investigates the consensus control of second-order multi-agent systems in the presence of stationary obstacles. Every agent is faced with unknown model uncertainties. Based on the finite-time control barrier function (FTCBF), a safety-certified consensus control method is proposed to achieve collision-free consensus formation. First, a nominal virtual control law is designed to generate the predefined formation. An FTCBF-based state set is constructed to guarantee the safety of multi-agent systems. Next, a finite-time observer with robust exact differentiator is devised to recover unknown model uncertainties. By using the estimated information, we develop an anti-disturbance control law to track the optimal signal. Finally, error signals of the closed-loop system are proved to be uniformly ultimately bounded, and the multi-agent system can be ensured to be safety-certified. A simulation example is employed to demonstrate the effectiveness of the proposed safety-certified consensus control method.

Key Words: Multi-agent system, consensus control, finite-time control barrier function, finite-time observer

1 Introduction

Over the past years, leader-following consensus control of multi-agent systems has aroused increasing attention from various communities and been applied in many fields, such as unmanned aerial vehicles, unmanned mobile robots, unmanned surface vehicles, and unmanned underwater vehicles [1–6]. Different from leader-less consensus control [7–9], leader-following consensus control aims to drive all controlled objects to achieve a common motion in the presence of leaders under the distributed communication. [9–13]. Leader-following consensus control guide by multiple leaders is also considered as containment control [14–16]. According to the property of leaders, leader-following control can be further divided into containment tracking guided by time-varying trajectories [9–11]and containment maneuvering guided by parameterized paths [12, 13]. Note that these existing containment control methods are under ideal safety conditions [9–16].

Safety is an important requirement when controlling multi-agent systems. Safety conditions include collision avoidance, connectivity preservation, communication delay, states and input constraints. Control barrier functions (CBFs) have been proposed as an efficient tool to design safety-critical controllers [17–19]. The key point of CBFs is to map constrained states onto constrained inputs by means of forwarding invariance of sets under safety conditions. Recently, many CBFs-based safety-critical control methods have been proposed for nonlinear systems [17–19]. However, these CBFs-based safety-critical control methods are centralized.

Motivated by the above observations, we propose a safety-certified distributed control method for leader-following consensus of a class of second-order nonlinear multi-agent systems based on finite-time control barrier function (FTCBF). The dynamics of followers is guided by a parameterized path and in the presence of unknown nonlinearities and stationary obstacles. At first, we design a nominal virtual control law to generate the predefined formation. Next, we construct an FTCBF-based state constraint set to guarantee safety among multi-agent systems. Then, a Levant finite-time observer is utilized to recover unknown model uncertainties. Finally, an anti-disturbance control law is constructed by using the estimated information. All error signals in the closed-loop is proved to be uniformly ultimately bounded, and the safety of multi-agent systems is guaranteed. In contrast to the existing leader-following consensus control methods in [9–16], the proposed consensus control method herein is not only collision-free but also finite-time safety-critical.

This paper is organized as follows. Section II gives problem formulation. Section III presents the controller design. Section IV shows the stability analysis. Section V provides the simulation results to demonstrate the effectiveness of the proposed method. Section V concludes this paper.

2 Preliminaries and Problem formulation

2.1 Notations

Define the following notations used in this paper. $\mathcal{R}$, $\mathcal{R}^+$, and $\mathcal{R}^n$ is a real set, a positive real set, and an n-dimensional vector set, respectively. $\|\cdot\|$ denotes the Euclidean norm of a vector. For $\boldsymbol{s} = [s_1, \cdots, s_n]^T \in \mathbb{R}^n$ and $0 < \iota < 1$, $\text{sig}^\iota(\boldsymbol{s})$ is defined as $\text{sig}^\iota(\boldsymbol{s}) = [\text{sign}(s_1)|s_1|^\iota, ..., \text{sign}(s_n)|s_n|^\iota]^T$, where $\text{sign}(\cdot)$ is a signum function.

This paper is supported in part by the Hainan Special PhD Scientific Research Foundation of Sanya Yazhou Bay Science and Technology City (HSPHDSRF-2022-01-007), and in part by the Key R&D Program of Guangdong (2020B1111010002), in part by the Hainan Province Science and Technology Special Fund (ZDYF2021GXJS041), and in part by the National Natural Science Foundation of China (U2141234).

2.2 Problem formulation

Consider a multi-agent system containing N second-order agents. Here, the dynamics of the ith agent is expressed as

$$\begin{aligned} \dot{x}_{i1}(t) &= x_{i2}(t), \\ \dot{x}_{i2}(t) &= f_i(x_i, t) + u_i(t), \end{aligned} \tag{1}$$

where $x_{i1}(t) \in \mathcal{R}^2$ and $x_{i2}(t) \in \mathcal{R}^2$ are the state of the agent with $x_i(t) = [x_{i1}^T(t), x_{i2}^T(t)]^T$. $f_i(x_i, t) \in \mathcal{R}^2$ denotes the unknown model uncertainty. $u_i(t) \in \mathcal{R}^2$ presents the actual control input.

Consider a virtual leader moving along the smooth parametrized path $x_0(\theta) \in \mathcal{R}^2$, where θ is a parameter variable. A communication graph $\mathcal{G}$ is defined as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with $\mathcal{V}$ and $\mathcal{E}$ being a node and an edge set, respectively. $\mathcal{V} = \{\mathcal{V}^F, \mathcal{V}^L\}$ is the set of agents and the virtual leader. $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$ is used to depict the communication links of agents and the virtual leader. The following assumptions are used in this paper.

Assumption 1: The first-order partial derivative of the given signal $x_0(\theta)$ is bounded and satisfies $\|x_0^\theta\| \le \bar{x}_0^\theta$ with $x_0^\theta = \partial x_0/\partial\theta$ and $\bar{x}_0^\theta \in \mathcal{R}^+$.

Assumption 2: For the communication topology of multiple agents and one virtual leader, there is at least one agent that can obtain the information of the virtual leader.

The safety-certified consensus control of second-order multi-agent system need to satisfy the following objectives.

- Consensus objective: Drive each agent to follow the virtual leader and hold a predefined formation such that

$$\lim_{t\to\infty} \|x_{i1}(t) - x_0(\theta(t)) - x_{i0}\| \le \delta_1, \tag{2}$$

  where $x_{i0} \in \mathcal{R}^2$ is a predefined deviation from the virtual leader. δ_1 is a positive constant.
- Dynamic objective: Force the parameter dynamics $\dot{\theta}(t)$ to converge to the expected speed such that

$$\lim_{t\to\infty} |\dot{\theta}(t) - \nu_s| \le \delta_2, \tag{3}$$

  where $\nu_s \in \mathcal{R}$ is a expected updated speed. $\delta_2 > 0$ is a small constant.
- Safety objective: Ensure no collision between agents and static obstacles such that

$$\|x_{i1} - x_o\| \ge D_s + \varrho_o, \quad o = 1, ..., N_o, \tag{4}$$

  where $x_o \in \mathcal{R}^2$ is the position of static obstacles. $N_o > 0$ is the total number of obstacles. $\varrho_o > 0$ denotes the radius of the oth obstacle. $D_s > 0$ stands for the preset safe distance.

3 Controller design

In this section, a distributed control law is designed to achieve the consensus formation. To ensure no collisions between agents and static obstacles, a safety-certified region and an admissible state set are constructed by using the finite-time control barrier function (FTCBF). Based on robust exact differentiator (RED), a finite-time observer is devised to estimate unknown uncertainties of agents. Then, an actual control law is developed.

Step 1: Define a consensus tracking error e_{i1} as follows

$$\begin{aligned} e_{i1} = &\sum_{j=1}^{N} a_{ij}(x_{i1} - x_{j1} - x_{ij0}) \\ &+ a_{i0}(x_{i1} - x_0(\theta) - x_{i0}), \end{aligned} \tag{5}$$

where if the ith agent can receive the information from the jth agent, $a_{ij} = 1$, $i \ne j$, $j = 1, ..., N$; otherwise, $a_{ij} = 0$. If the ith agent can obtain the information from the virtual leader, $a_{i0} = 1$; otherwise, $a_{i0} = 0$. $x_{ij0} = x_{i0} - x_{j0}$.

Taking the derivative of e_{i1} along (1), one has

$$\begin{aligned} \dot{e}_{i1} =& d_i x_{i2} - \sum_{j=1}^{N} a_{ij} x_{j2} - a_{i0} x_0^\theta \dot{\theta}, \\ \dot{\theta} =& \nu_s - \vartheta, \end{aligned} \tag{6}$$

where $d_i = \sum_{i=1}^{N} a_{ij} + a_{i0}$.

To stabilize the error dynamics in (6), we design a distributed control law as follows

$$\alpha_i = \frac{1}{d_i}\left(-k_{i1}e_{i1} + \sum_{j=1}^{N} a_{ij}x_{j2} + a_{i0}x_0^\theta \nu_s\right), \tag{7}$$

where k_{i1} is a positive gain constant.

To coordinate agents and reference signal, the update law is designed as

$$\dot{\vartheta} = -\lambda\left(\vartheta + \varepsilon\sum_{j=1}^{N} a_{i0}(x_0^\theta)^T e_{i1}\right). \tag{8}$$

To avoid the collision between agents ans N_o static obstacles, a FTCBF $h_{io}(x_i)$ is constructed as follows

$$h_{io}(x_i) = \|x_{i1} - x_o\|^2 - \varrho_o^2, \tag{9}$$

where $o = 1, ..., N_o$.

Based on the designed FTCBF $h_{io}(x_i)$, a safety-certified motion region for the ith agent can be described as

$$\mathcal{C}_{io}(x_{i1}) = \{x_{i1} \in \mathcal{R}^2 : h_i(x_i) \ge 0\}. \tag{10}$$

To ensure that agents can go into the safety region in finite time, an admissible state space is developed as [20]

$$\mathcal{S}_{io}(x_{i2}) = \left\{x_{i2} \in \mathcal{R}^2 : \dot{h}_{io}(x_i) \ge -\zeta_i \mathrm{sgn}^{\epsilon_i}(h_{io})\right\}, \tag{11}$$

where $\zeta_i > 0$ and $0 < \epsilon_i < 1$.

In order to unify the tracking performance and safety requirement, a distributed quadratic optimization is employed as follows

$$\begin{aligned} x_{i2}^* = \arg \min_{x_{i2}\in\mathcal{R}^2} \quad & J_i(\alpha_i) = \|x_{i2} - \alpha_i\|^2 \\ \text{s.t.} \quad & x_{i2} \in \mathcal{S}_{io}(x_{i2}), \quad o = 1, ..., N_o. \end{aligned} \tag{12}$$

Noting that the quadratic optimization with multiple constraints can be solved by neurodynamic optimization such as projection neural network [21] and recurrent neural network [22]. It can obtain $\|x_{i2}^* - \alpha_i\| \le \bar{\alpha}$ with $\bar{\alpha}$ being a positive constant.

Step 2: To acquire the smooth property and the derivative of x_{i2}^*, a linear tracking differentiator (LTD) is devised as

$$\begin{aligned}\dot{q}_{if} &= q_{if}^d,\\ \dot{q}_{if}^d &= -\mu_i^2(q_{if} - x_{i2}^*) - 2\mu_i q_{if}^d,\end{aligned} \tag{13}$$

where q_{if} and q_{if}^d represents the estimated information of x_{i2}^* and $\dot{x}_{i2}^*$. According to the stability analysis from [23], it can be concluded that the filtered signals q_{if} and q_{if}^d can converge to the neighbor region of x_{i2}^* and $\dot{x}_{i2}^*$, i.e., satisfying $\|q_{if} - x_{i2}^*\| \le \bar{q}_f$ and $\|q_{if}^d - \dot{x}_{i2}^*\| \le \bar{q}_f^d$ with $\bar{q}_f$ and $\bar{q}_f^d$ being positive constants.

Step 3: Define the estimated errors $\tilde{x}_{i2} = \hat{x}_{i2} - x_{i2}$ and $\tilde{f}_i = \hat{f}_i - f_i$, where $\hat{x}_{i2}$ and $\hat{f}_i$ are the estimations of x_{i2} and f_i, respectively. Then, a finite-time observer based on robust exact differentiator (RED) is developed as follows [24]

$$\begin{aligned}\dot{\hat{x}}_{i2} &= -\gamma_{i1} L_i^{\frac{1}{2}} \mathrm{sgn}^{\frac{1}{2}}(\tilde{x}_{i2}) + \hat{f}_i + u_i,\\ \dot{\hat{f}}_i &= -\gamma_{i2} L_i \mathrm{sgn}^0(\tilde{x}_{i2}),\end{aligned} \tag{14}$$

where γ_{i1} and γ_{i2} are the designed positive constants. L_i is a scaling factor. It is assumed that the first-order derivative of f_i is bounded. According to [25], the first-order observer (14) is finite-time stable, which means that the estimated error $\tilde{f}_i$ is bounded and satisfies $\|\tilde{f}_i\| \le \bar{f}_i \in \mathcal{R}^+$.

Letting the tracking error $e_{i2} = x_{i2} - q_{if}$, we differentiate it along (1) as

$$\dot{e}_{i2} = f_i + u_i - q_{if}^d. \tag{15}$$

To converge the error e_{i2}, a observer-based control law is designed as follows

$$u_i = -k_{i2} e_{i2} - \hat{f}_i + q_{if}^d - d_i e_{i1}. \tag{16}$$

where k_{i2} is the designed positive constant.

4 Safety and Stability analysis

The safety analysis of multi-agent systems is given by the following lemma. *Lemma 1:* Given a second-order agent expressed as (1), if the state variable x_{i2} belongs to the safety-certified state set defined by (11) for all agents and $x_{i2} \in \mathcal{S}_{io}(x_{i2})$, $i = 1, ..., N$, the closed-loop multi-agent system is safety-certified.

Proof: According to [21, 22], the state x_{i2} can converge to the optimal solution x_{i2}^* and meet the collision-free condition. From [20], it gets that the set $\mathcal{C}_{io}(x_{i1})$ is forward invariant by using the optimal state x_{i2}^*, i.e. the set $\mathcal{C}_{io}(x_{i1})$ is safe. It also shows that the agent state $x_{i1}(t)$ will stay in the set $\mathcal{C}_{io}(x_{i1})$ for $t > t_0$ if it satisfies $x_{i1}(t_0) \in \mathcal{C}_{io}(x_{i1})$, $i = 1, ..., N$. Thus, it is ensured that the closed-loop multi-agent systems is safety certified.

Recalling error dynamics of the closed-loop system by using the equations (6), (7), (8), (15) and (16), we have

$$\begin{aligned}\dot{e}_{i1} &= -k_{i1} e_{i1} + d_i z_i + a_{i0} x_0^\theta \vartheta,\\ \dot{e}_{i2} &= -k_{i2} e_{i2} - \tilde{f}_i - d_i e_{i1},\\ \dot{\vartheta} &= -\lambda \left(\vartheta + \varepsilon \sum_{i=1}^N a_{i0} (x_0^\theta)^T e_{i1} \right).\end{aligned} \tag{17}$$

where $z_i = x_{i2} - \alpha_i$.

The stability of the closed-loop system is shown via the following theorem.

Theorem 1: Consider the multi-agent system expressed as (1) in presence to multiple static obstacles with the distributed control law (7), the parameter update law (8), the admissible state space (11), the optimal control law (12), the LTD (13), the Levant observer (14) and the actual control control law (16). Under Assumptions 1 and 2, all error signals of the proposed closed-loop system are uniformly ultimately bounded, and the multi-agent system is safe.

Proof: Construct a candidate Lyapunov function as

$$V = \frac{1}{2} \sum_{i=1}^N (e_{i1}^T e_{i1} + e_{i2}^T e_{i2}) + \frac{\vartheta^2}{2\lambda\varepsilon}. \tag{18}$$

Taking the time derivative of V along the dynamics (17), one has

$$\begin{aligned}\dot{V} = &\sum_{i=1}^N \Big(e_{i1}^T(-k_{i1} e_{i1} + d_i z_i + a_{i0} x_0^\theta \vartheta) + e_{i2}^T(-\tilde{f}_i\\ &-k_{i2} e_{i2} - d_i e_{i1})\Big) - \frac{\vartheta}{\varepsilon} \left(\vartheta + \varepsilon \sum_{i=1}^N a_{i0} (x_0^\theta)^T e_{i1} \right)\\ = &\sum_{i=1}^N \Big(-k_{i1}\|e_{i1}\|^2 - k_{i2}\|e_{i2}\|^2 + d_i e_{i1}^T z_i\\ &-e_{i2}^T \tilde{f}_i - d_i e_{i2}^T e_{i1} \Big) - \vartheta^2/\varepsilon.\end{aligned} \tag{19}$$

Since $z_i = e_{i2} + q_{if} - x_{i2}^* + x_{i2}^* - \alpha_i$, it renders

$$\begin{aligned}\dot{V} \le &\sum_{i=1}^N \Big(-k_{i1}\|e_{i1}\|^2 - k_{i2}\|e_{i2}\|^2 + \bar{f}_i \|e_{i2}\|\\ &+d_i(\bar{\alpha} + \bar{q}_f)\|e_{i1}\| \Big) - \vartheta^2/\varepsilon.\end{aligned} \tag{20}$$

Let $c_1 = \min\{k_{i1}, k_{i2}\}$, $c_2 = \max\{d_i(\bar{\alpha} + \bar{q}_f), \bar{f}_i\}$ $k = 1, ..., N$. The derivative of V can be rewritten as

$$\begin{aligned}\dot{V} \le &- c_1 \sum_{i=1}^N (\|e_{i1}\|^2 + \|e_{i2}\|^2) - \vartheta^2/\varepsilon\\ &+ c_2 \sum_{i=1}^N (\|e_{i1}\| + \|e_{i2}\|)\end{aligned} \tag{21}$$

Define $E = [e_1^T, e_2^T, \vartheta]^T$ with $e_1 = [e_{11}, ..., e_{N1}]^T$ and $e_2 = [e_{12}, ..., e_{N2}]^T$. The inequality (21) is presented as

$$\dot{V} \le -c_1^{'} \|E\|^2 + c_2 \|E\|, \tag{22}$$

where $c_1^{'} = \min\{c_1, 1/\varepsilon\}$.

For $\|E\| \ge c_2/(c_1^{'} \varrho)$, it can obtain

$$\dot{V} \le -c_1^{'}(1 - \varrho)\|E\|^2. \tag{23}$$

Therefore, it can be concluded that all errors of the closed-loop system is uniformly ultimately bounded, and it satisfies

$$\|E(t)\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \max \left\{ \|E(t_0)\| e^{-\beta(t - t_0)}, \frac{c_2}{c_1^{'} \varrho} \right\} \tag{24}$$

for $t \ge t_0$, where $P = \mathrm{diag}\{1, 1/\lambda\epsilon\}$ and $\beta = 2c_1^{'}(1 - \varrho)/\lambda_{\max}(P)$.

5 Simulation results

In this section, simulation results are provided to demonstrate the effectiveness of safety-certified consensus control method. One virtual leader, five agents and four circular obstacles are employed in this simulation, and Fig. 1 describes their communication topology.

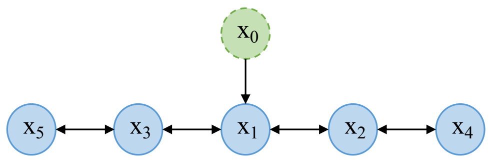

Fig. 1: The safety-certified containment formation.

In this simulation, uncertainty terms of these agents are set as $f_i = [0.2\sin(x_{i1,1})\cos(x_{i2,1}), 0.4\sin(\pi/3 + x_{i1,2})\cos(x_{i2,2})]^T$, $i = 1, ..., 5$. Their initial status are set as $x_1 = [-2, 0, 0, 0]^T$, $x_2 = [-4, -4, 0, 0]^T$, $x_3 = [-6, -8, 0, 0]^T$, $x_4 = [-8, -12, 0, 0]^T$, $x_5 = [-10, -16, 0, 0]^T$, respectively. We select the position and radius of static circular obstacles as $x_1 = [12, 18]^T$, $x_2 = [30, 32]^T$, $x_3 = [50, 46]^T$, $\rho_1 = 2$, $\rho_2 = 3$, $\rho_3 = 4$, respectively. Parameters of the proposed method are selected as $x_{10} = [0, 0]^T$, $x_{20} = [-8, 0]^T$, $x_{30} = [0, -8]^T$, $x_{40} = [-16, 0]^T$, $x_{50} = [0, -16]^T$, $D_s = 1$, $\nu_s = 0.5$, $k_{i1} = 0.8$, $\lambda = 10$, $\varepsilon = 10$, $\zeta_i = 1$, $\epsilon_i = 0.8$, $\mu_i = 10$, $\gamma_{i1} = 2.12$, $\gamma_{i2} = 1.1$, $L_i = 3.5$, $k_{i2} = 10$.

Simulation results are shown in Fig. 2 - Fig. 5. Specifically, Fig. 2 depicts the actual trajectories of five agents in the presence of three circular obstacles. It can also be seen that every agent avoids all static obstacles while trying to hold consensus with other agents. The consensus tracking errors of all agents are presented by Fig. 3, which reflects that errors are able to converge to small neighbors of the origin in finite time. Fig. 4 describes the varying curves of FTCBFs of five agents. It can be concluded that the multi-agent systems is safety-certified. Fig. 5 gives the real uncertainties and the estimated ones from the proposed observer (14). It shows that the RED-based observer is capable to recover the unknown uncertainties.

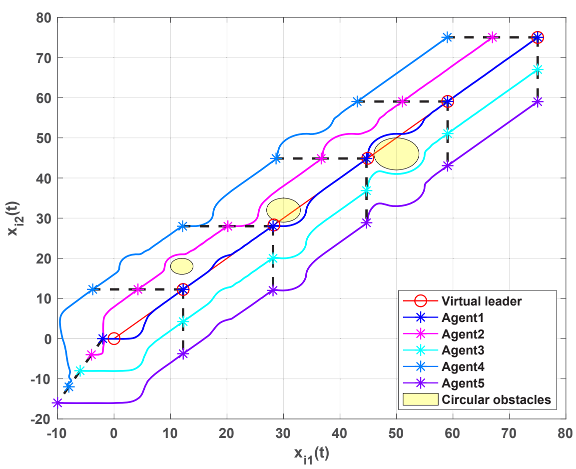

Fig. 2: The safety-certified containment formation.

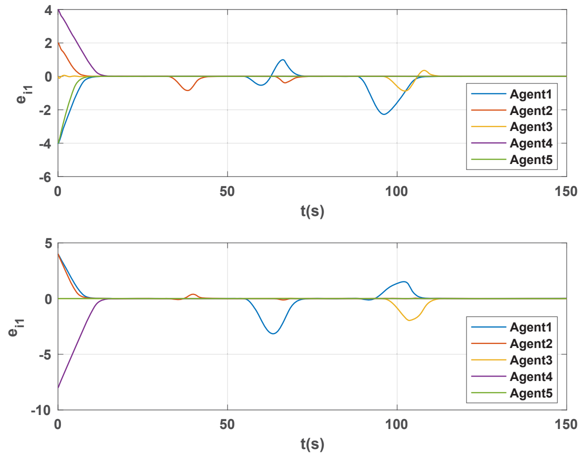

Fig. 3: The tracking errors of five agents.

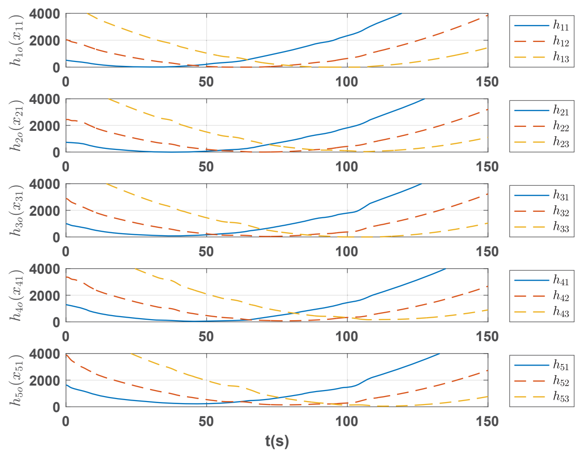

Fig. 4: The varying curve of finite-time control barrier function of five agents.

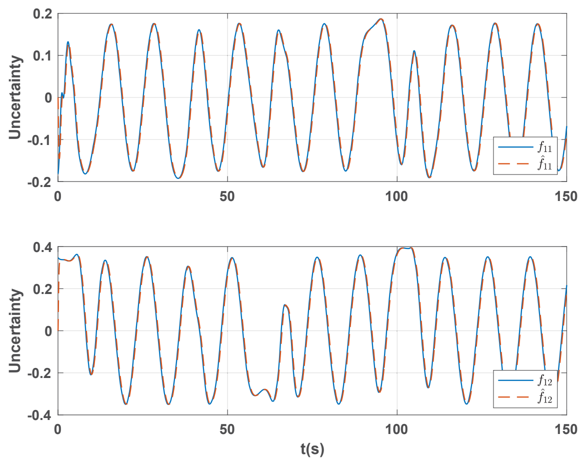

Fig. 5: The model uncertainties of agent 1.

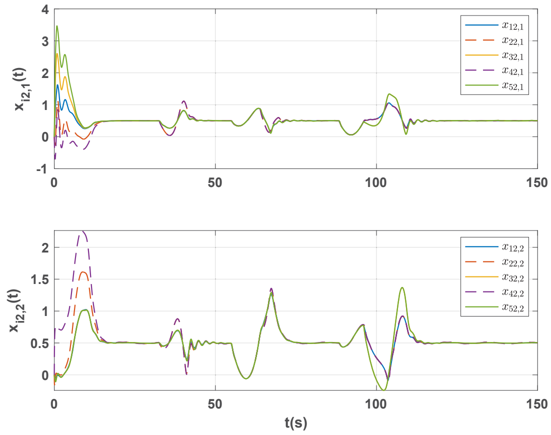

Fig. 6: The actual state of five agents.

6 Conclusion

This paper investigated the safety-certified consensus control of second-order multi-agent systems in presence of stationary obstacles. Each agent is also faced with the model uncertainties. First, a distributed virtual control law is proposed based on backstepping principle. The constructed FTCBF can guarantee the safety of multi-agent systems. Next, a finite-time observer is devised to recover unknown model uncertainties. Finally, all errors of the closed-loop system are proved to be uniformly ultimately bounded, and the safety of multi-agent systems is able to be ensured. Simulation results demonstrate the effectiveness of the proposed safety-certified consensus control method.

References

[1] Z. Peng, J. Wang, D. Wang, and Q. Han, "An overview of recent advances in coordinated control of multiple autonomous surface vehicles," *IEEE Transactions on Industrial Informatics*, vol. 17, no. 2, pp. 732–745, 2021.

[2] W. Xie, D. Cabecinhas, R. Cunha, and C. Silvestre, "Adaptive backstepping control of a quadcopter with uncertain vehicle mass, moment of inertia, and disturbances," *IEEE Transactions on Industrial Electronics*, vol. 69, no. 1, pp. 549–559, 2021.

[3] L. Qiao and W. Zhang, "Trajectory tracking control of auvs via adaptive fast nonsingular integral terminal sliding mode control," *IEEE Transactions on Industrial Informatics*, vol. 16, no. 2, pp. 1248–1258, 2019.

[4] W. Xie, D. Cabecinhas, R. Cunha, and C. Silvestre, "Robust motion control of an underactuated hovercraft," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 5, pp. 2195–2208, 2018.

[5] W. Wu, Z. Peng, D. Wang, L. Liu, and Q.-L. Han, "Network-based line-of-sight path tracking of underactuated unmanned surface vehicles with experiment results," *IEEE Transactions on Cybernetics*, pp. 1–11, 2021.

[6] W. Wu, Z. Peng, L. Liu, and D. Wang, "A general safety-certified cooperative control architecture for interconnected intelligent surface vehicles with applications to vessel train," *IEEE Transactions on Intelligent Vehicles*, 2022, accepted.

[7] G. H. Wen, Z. S. Duan, W. W. Yu, and G. R. Chen, "Consensus in multi-agent systems with communication constraints," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 2, pp. 170–182, 2012.

[8] Z. Li, G. Wen, Z. Duan, and W. Ren, "Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs," *IEEE Transactions on Automatic Control*, vol. 60, no. 4, pp. 1152–1157, 2015.

[9] Y. Yang, Y. Li, D. Yue, Y.-C. Tian, and X. Ding, "Distributed secure consensus control with event-triggering for multiagent systems under dos attacks," *IEEE Transactions on Cybernetics*, vol. 51, no. 6, pp. 2916–2928, 2021.

[10] W. Wang, D. Wang, Z. Peng, and T. Li, "Prescribed performance consensus of uncertain nonlinear strict-feedback systems with unknown control directions," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 9, pp. 1279–1286, Sep 2016.

[11] J. Zhou, Y. Lv, G. Wen, and X. Yu, "Resilient consensus of multiagent systems under malicious attacks: Appointed-time observer-based approach," *IEEE Transactions on Cybernetics*, in press, 2021, DOI:10.1109/TCYB.2021.3058094.

[12] Y. Zhang, D. Wang, and Z. Peng, "Consensus maneuvering for a class of nonlinear multivehicle systems in strict-feedback form," *IEEE Transactions on Cybernetics*, vol. 49, no. 5, pp. 1759–1767, 2019.

[13] Z. Peng, J. Wang, and D. Wang, "Distributed maneuvering of autonomous surface vehicles based on neurodynamic optimization and fuzzy approximation," *IEEE Transactions on Control Systems Technology*, vol. 26, no. 3, pp. 1083–1090, May 2018.

[14] Z. Peng, J. Wang, and D. Wang, "Containment maneuvering of marine surface vehicles with multiple parameterized paths via spatial-temporal decoupling," *IEEE/ASME Transactions on Mechatronics*, vol. 22, no. 2, pp. 1026–1036, Apr 2017.

[15] ——, "Distributed containment maneuvering of multiple marine vessels via neurodynamics-based output feedback," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 4, pp. 3831–3839, May 2017.

[16] Y. Zhang, D. Wang, Z. Peng, and T. Li, "Distributed containment maneuvering of uncertain multiagent systems in mimo strict-feedback form," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 2, pp. 1354–1364, 2021.

[17] W. Xiao and C. Belta, "Control barrier functions for systems with high relative degree," pp. 474–479, 2019.

[18] S. Kolathaya and A. D. Ames, "Input-to-state safety with control barrier functions," *IEEE Control Systems Letters*, vol. 3, no. 1, pp. 108–113, 2019.

[19] C. Dong, Y. Liu, and Q. Wang, "Barrier lyapunov function based adaptive finite-time control for hypersonic flight vehicles with state constraints," *ISA Transactions*, vol. 96, pp. 163–176, 2020.

[20] M. Srinivasan, S. Coogan, and M. Egerstedt, "Control of multi-agent systems with finite time control barrier certificates and temporal logic," in *2018 IEEE Conference on Decision and Control (CDC)*, 2018, pp. 1991–1996.

[21] Y. Xia, "An extended projection neural network for constrained optimization," *Neural Computation*, vol. 16, no. 4, pp. 863–883, 2004.

[22] G. Li, Z. Yan, and J. Wang, "A one-layer recurrent neural network for constrained nonsmooth invex optimization," *Neural Networks*, vol. 50, pp. 79–89, 2014.

[23] B.-Z. Guo and Z.-L. Zhao, "On convergence of tracking differentiator," *International Journal of Control*, vol. 84, no. 4, pp. 693–701, Apr 2011.

[24] W. Wu, Z. Peng, D. Wang, L. Liu, and G. Nan, "Anti-disturbance leader-follower synchronization control of marine vessels for underway replenishment based on robust exact differentiators," *Ocean Energineering*, 2022.

[25] A. Levant, "Homogeneity approach to high-order sliding mode design," *Automatica*, vol. 41, no. 5, pp. 823–830, 2005.